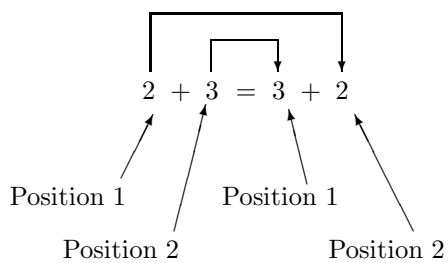
Chapter 1

The Commutative, Associative, and Distributive Laws/Properties

1.1 The Commutative Property of Addition and Multiplication



2 commuted (traveled) from position 1 to position 2

and

3 commuted (traveled) from position 2 to position 1.

(A commuter is a traveler. Do not say "communitive" property)

The picture illustrates the **commutative property of addition**.

In general

$$\boxed{\text{a+b=b+a}}$$

where a and b are any real numbers (like -6.4, $\frac{2}{7}$, π). Any real number can hide in the a-box or the b-box.

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Is subtraction commutative?

Is $3 - 1 = 1 - 3$ a true statement?

No, because $3-1-2$ and $1-3 = -2$. If we find one counterexample, one example that shows that subtraction is not commutative, the general property (using a and b) does not exist.

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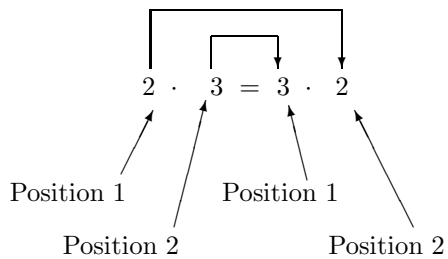

2 commuted (traveled) from position 1 to position 2

and

3 commuted (traveled) from position 2 to position 1.

The picture illustrates the **commutative property of multiplication**.

In general

$$\boxed{\text{ab=ba}}$$

where a and b are any real numbers (like -6.4, $\frac{2}{7}$, π). Any real number can hide in the a-box or the b-box.

Note that ab means a times b.

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Is division commutative?

Is $4 \div 2 = 2 \div 4$ a true statement?

No, because $4 \div 2 = 2$ and $2 \div 4 = \frac{1}{2} = 0.5$. If we find one counterexample, one example that shows that division is not commutative, the general property (using a and b) does not exist.

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1.2 The Associative Property of Addition and Multiplication

$$5 + \underbrace{(4 + 3)}_{\substack{\text{4 is associated} \\ \text{(grouped) with 3}}} = \underbrace{(5 + 4)}_{\substack{\text{4 is associated} \\ \text{(grouped) with 5}}} + 3$$

$5 + (4 + 3) = 5 + 7 = 12$ and $(5 + 4) + 3 = 9 + 3 = 12$ also.

The picture illustrates the **Associative property of addition**.

In general

$$\boxed{\text{a+(b+c)=(a+b)+c}}$$

where a and b are any real numbers.

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Is subtraction associative?

Is $5 - (4 - 3) = (5 - 4) - 3$ a true statement?

No, because $5 - (4 - 3) = 5 - 1 = 4$ and $(5 - 4) - 3 = 1 - 3 = -2$. The general property (using a and b) does not exist.

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$$5 \cdot \underbrace{(4 \cdot 3)}_{\substack{\text{4 is associated} \\ \text{(grouped) with 3}}} = \underbrace{(5 \cdot 4)}_{\substack{\text{4 is associated} \\ \text{(grouped) with 5}}} \cdot 3$$

$5(4 \cdot 3) = 5(12) = 60$ and $(5 \cdot 4) \cdot 3 = (20)3 = 60$ also.

The picture illustrates the **Associative Property of Multiplication**.

In general

$$\boxed{a(bc)=(ab)c}$$

where a and b are any real numbers.

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Is division associative?

Is $16 \div (4 \div 2) = (16 \div 4) \div 2$ a true statement?

No, because $16 \div (4 \div 2) = 16 \div 2 = 16 \div 2 = 8$ and $(16 \div 4) \div 2 = (4) \div 2 = 2$. Division is not associative, the general property (using a and b) does not exist.

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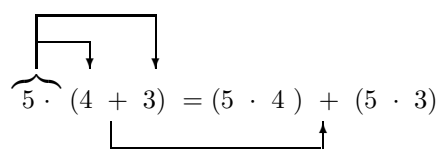
1.3 The Distributive property of Multiplication over Addition and/or Subtraction



$5(4+3) = 5(7) = 35$ and $(5)(4) + (5)(3) = 20 + 15 = 35$ also.

The picture illustrates the **Distributive property of multiplication over addition**.

In general

$$\boxed{a(b+c)=ab+ac}$$

where a and b are any real numbers.

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$$\overbrace{5\cdot}\ (4\ -\ 3)\ =(5\ \cdot\ 4\ )\ -\ (5\ \cdot\ 3)$$

$5(4-3)=5(1)=5$ and $(5)(4)-(5)(3)=20-15=5$ also.

The picture illustrates the **Distributive property of multiplication over Subtraction**.

In general

$$\boxed{\text{a(b+c)=ab+ac}}$$

where a and b are any real numbers.

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$$\overbrace{5\cdot}\ (4\ \cdot\ 3)\ =(5\ \cdot\ 4\ )\ \cdot\ (5\ \cdot\ 3)$$

$5(4\cdot 3)=5(12)=60$ and $(5)(4)\cdot(5)(3)=20\cdot 15=300$ which is different.

The example shows that the distributive property does not apply to multiplication over multiplication.

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$$\overbrace{16\div}\ (4\ \div\ 2)\ =(16\ \div\ 4\ )\ \div\ (16\ \div\ 2)$$

$16\div(4\div 2)=16\div 2=8$ and $(16\div 4)\div(16\div 2)=4\div 8=\dfrac{1}{2}$ which is different.

The example shows that the distributive property does not apply to division over division.

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1.4 Factoring the GCF

The distributive property of multiplication over addition/subtraction can be applied in reverse.

$a(b \pm c) = ab \pm ac$ implies $ab \pm ac = a(b \pm c)$.

FDactoring is the art of taking a sum (addition of terms) or difference (subtraction of terms) into a product (multiplication of factors).

Example :

Factor $15x + 20y$.

Solution:

$$\begin{aligned} 15x + 20y &= 3 \cdot 5x + 4 \cdot 5y \\ &= 5(3x + 4y) \end{aligned}$$

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1.5 Exercise 2

State the property or indicate that no property is applicable.

1. $5 + x = x + 5$
2. $7(a - 2) = 7a - 7(2)$
3. $5(x \cdot 2) = (5x)(2)$
4. $(y + 2) + 4 = 4 + (y + 2)$
5. $6 \cdot (t \cdot 2) = (6t) \cdot (6 \cdot 2)$
6. $w - 5 = 5 - w$
7. $xy = yx$
8. $a\pi = \pi a$
9. $(-4.7) + [(6.5) + (\sqrt{2})] = [(-4.7) + (6.5)] + (\sqrt{2})$
10. $x - 4 = 4 - x$
11. Factor $12x + 18$
12. Factor $24x^2 + 12x$

$\boxed{\text{STOP!}}$

The solutions follow:

Keep them covered up till you have worked out each of the problems above.

1. $5 + x = x + 5$
   Solution:
   $5 + x = x + 5$ illustrates the commutative property of addition. The terms commuted to each other's position.

2. $7(a - 2) = 7a - 7(2)$
   Solution:
   $7(a - 2) = 7a - 7(2)$ illustrates the distributive property of multiplication over addition.

3. $5(x \cdot 2) = (5x)(2)$
   Solution:
   $5(x \cdot 2) = (5x)(2)$ illustrates the associative property of multiplication.

4. $(y + 2) + 4 = 4 + (y + 2)$
   Solution:
   $(y+2)+4 = 4+(y+2)$ illustrates the commutative property of addition. $(y+2)$ and 4 have commuted to each other's position. y remains grouped with 2.

5. $6 \cdot (t \cdot 2) = (6t) \cdot (6 \cdot 2)$
   Solution:
   $6 \cdot (t \cdot 2) = (6t) \cdot (6 \cdot 2)$ does not illustrate any of the mentioned properties. You cannot distribute multiplication over multiplication.

6. $w - 5 = 5 - w$
   Solution:
   $w - 5 = 5 - w$ does not illustrate any of the mentioned properties. Subtraction is not commutative. We shall learn later that $w - 5 = w + (-5) = (-5) + w$ because addition is commutative.

7. $xy = yx$
   Solution:
   $xy = yx$ illustrates the commutative property of multiplication. The terms commuted to each other's position.

8. $a \div \pi = \pi \div a$
   Solution:
   $a \div \pi = \pi \div a$ does not illustrate any of the mentioned properties. Division is not commutative.

9. $(-4.7) + [(6.5) + (\sqrt{2})] = [(-4.7) + (6.5)] + (\sqrt{2})$
   Solution:
   $(-4.7) + [(6.5) + (\sqrt{2})] = [(-4.7) + (6.5)] + (\sqrt{2})$ illustrates the associative property of addition. 6.5 is associated (grouped) with $\sqrt{2}$ and -4.7.

10. $x - 4 = 4 - x$
    Solution:
    $x - 4 = 4 - x$ does not illustrate any of the mentioned properties. Subtraction is not commutative.

11. Factor $12x + 18$ **Solution:**
    $$\begin{aligned} 12x + 18 &= (3)(4)x + (3)(6) \\ &= 3(4x + 6) \end{aligned}$$
    A quick check: $3(4x + 6) = 3(4x) + 3(6) = 12x + 18$

12. Factor $24x^2 + 12x$ **Solution:**

$$\begin{aligned} 24x^2 + 12x &= 2(12)x\ x + 1(12)x \\ &= 12x(2x+1) \end{aligned}$$

A quick check: $12x(2x+1) = 12x(2x) + 12x(1) = 24x^2 + 12x$